Expansionary Fiscal Contractions and Equilibrium Indeterminacy: A Case Study for Germany

by

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Abstract

At several occasions, the 1980's witnessed non-keynesian effects of fiscal contractions. Most notably, cuts in government spending in Ireland, Denmark and Germany are known to have coincided with increases in private consumption spending. Self-fulfilling expectations about the effects of stabilization policies may explain these and a diversity of opposing experiences. This paper formulates a real business cycle model with equilibrium indeterminacy, which allows for sunspot fluctuations as well as for systematic consumption effects of government policy in either direction. Cointegration tests and Euler equation estimates suggest that the model is approximately in accord with German data. Parameter estimates imply that the non-keynesian experiences are not due to self-fulfilling expectations but to the productivity effects of government-provided infrastructure.

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1. Introduction

A number of European countries have abandoned the road of keynesian economic policies in the 1980s, basically on the grounds of disillusion about the effects of deficit spending policies in the 1970s. Giavazzi and Pagano (1990) illustrate the turnaround in public spending programs for major OECD countries. They point out that the effects of fiscal contractions vary substantially in cross-country samples, with Denmark (1983-1984) and Ireland (1987-1989) the front-runners on the non-keynesian side, i.e. experiencing positive private consumption growth along with cuts in public spending, and Ireland (!) (1981-1984) and Spain (1986-1989) the leading examples of keynesian effects, where public spending and private consumption are positively correlated.

As a possible explanation Giavazzi and Pagano refer to a "German view" of government stabilization policies as articulated by Hellwig und Neumann (1987), who state that

"the direct impact of slower public expenditure growth is clearly negative... The indirect effect on aggregate demand of the initial reduction in expenditure growth occurs through an improvement in expectations if the measures taken are understood to be part of a credible medium-run program of consolidation, designed to permanently reduce the share of government in GDP...[and thus] taxation in the future."

According to this view, benign effects of fiscal consolidation rely on the perceived persistence of public spending cuts. If government spending is expected to be permanently lower, then permanent income rises and the thereby induced rise in private consumption may outweigh the depressing effects of elementary keynesian multiplier mechanics. If, however, government spending cuts are not long-run credible, then the latter effects dominate and hence keynesian results emerge. Giavazzi and Pagano (1990, 1996) argue that the credibility of fiscal contractions may rise with the severity of the measures taken, i.e. large spending cuts are more likely to result in increased private consumption than small.

Giavazzi and Pagano do not analyze the issue in a structural model; their approach is entirely confined to reduced forms. In an on first sight unrelated line of research, real business cycle (RBC) analysts, however, have established results that might prove helpful in explaining non-keynesian phenomena of the said type. For a major focus of RBC theory has recently been on models with equilibrium indeterminacy, see, e.g., Benhabib and Farmer (1994) and Farmer and Guo (1994). In such models convergence paths to the steady state are not unique, such that expectations can be self-fulfilling.

For instance, assume that people increase their consumption because they believe that a certain government policy will raise their permanent incomes, while in fact the government policy does not have real effects at all. In a standard RBC-model people will quickly realize that income does not respond to the government policy and thus consumption plans will be revised downwards. In an RBC-model with equilibrium indeterminacy, however, the initial rise in consumption will cause an increase in income which rectifies the initial beliefs, i.e. expectations are self-fulfilling. An economy of this type is said to exhibit sunspot fluctuations, i.e. fluctuations that do not depend on shocks to the fundamentals of the economy. Needless to say, shocks in fundamentals, such as government policies with real effects, may be reinforced by sunspot fluctuations.

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2 See, however, Bertola and Drazen (1993) for a theoretical model of expansionary fiscal contractions.
In this paper, I will analyze the scope for non-keynesian effects of fiscal contractions in the framework of RBC models with potential for equilibrium indeterminacy. I use an RBC model with production externalities which give rise to increasing returns on the aggregate level. The novel feature of this model as opposed to existing RBC models with equilibrium indeterminacy is the role of government expenditure: I assume the government to use tax proceeds for the production of infrastructure complementary to private production factors. This assumption serves two purposes: First, it motivates the existence of increasing returns in the aggregate despite constant returns for the individual producer, and second, it ensures a more realistic role of government activity than in standard RBC-models, where government expenditure is often assumed to yield zero utility, cf. Christiano and Eichenbaum (1992) or Johnson and Klein (1997)3.

While increasing returns are a necessary condition for equilibrium indeterminacy, they are not sufficient. Rather, equilibrium indeterminacy implies further restrictions on the deep parameters. I derive these following Wen's (1996) insightful analysis. I also derive the parameter conditions under which changes in government spending systematically increase or decrease private consumption. Depending on parameter magnitudes, the model thus allows for keynesian or for non-keynesian effects of fiscal contractions and either sort of effect can be due to systematic or to sunspot sources.

On the empirical side, I basically follow the estimation strategy laid out by Farmer and Guo (1995). I use German quarterly national accounts data to estimate the labor market Euler-equation by instrumental variables methods. In addition, I derive the long run equilibrium properties from the Euler-equations and check model adequacy by Johansen's (1988, 1995) method. Tests for the rank of the cointegration space as well as parameter estimates from the cointegrating vectors seem to indicate that the model is roughly in accord with the data. Based on the parameter estimates obtained I reject the possibility of non-keynesian sunspot fluctuations. Rather, my estimates suggest that non-keynesian effects of Germany's fiscal contraction in the 1980s emerged from systematic sources, in particular from a marginal benefit of infrastructure considerably lower than marginal tax rates.

The sequel of the paper is organized as follows: The theoretical model is introduced in section 2. Section 3 discuses the data and testable model implications. In section 4 Johansen tests for cointegration and parameter estimates are presented. Section 5 contains some concluding remarks.

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3The standard setup in which the only effect of government activity is a waste of resources is inconsistent with rational political economy considerations: Why should the public vote support government policies that are costly and useless?
2. The Model

Suppose an economy is composed of many identical agents who maximize expected utility under a number of constraints to be described below. Utility depends positively on consumption $C_t$ and negatively on hours worked $L_t$. The capital stock $K_t$ is predetermined so that the objective is to solve

$$
\max_{C_t, L_t, K_{t+1}, t \geq 0} U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u \left( C_t, \frac{L_t}{X_t} \right) \right], \quad (1)
$$

where $0 < \beta < 1$ is a discount factor and momentary utility $u$ is defined in consumption and labor units scaled by a growth component $X_t$. Scaling hours in such a way enables us to account for the negative growth trend found in German hours per capita. Similar preferences have been specified by Greenwood, Hercowitz and Huffman (1988) as well as Correia, Neves, and Rebelo (1995); the standard case of stationary hours is retrieved by setting $\nu = 0$. Note that we will expect $\nu$ to be negative in the case of negatively trended hours, i.e. for conventional momentary utility functions the marginal disutility of labor increases with the growth component.

Momentary utility is defined to be

$$
u \left( C_t, \frac{L_t}{X_t} \right) = \begin{cases} 
\ln C_t - \frac{1}{1 + \psi} \left( \frac{L_t}{X_t} \right)^{1+\psi} & \text{for } \psi \neq -1 \\
\ln C_t - \ln \left( \frac{L_t}{X_t} \right) & \text{for } \psi = -1
\end{cases}
$$

where $1/\psi$ is the intertemporal elasticity of substitution in labor. The model thus includes the Hansen (1985)-Rogerson (1988) case of indivisible labor, since utility is linear in labor for $\psi = 0$, i.e. for an infinite (aggregate) intertemporal elasticity of substitution.

The stock of public infrastructure $S_t$ declines with linear depreciation rate $0 < d \leq 1$ but increases with new government absorption $G_t$:

$$
S_t = (1-d)S_{t-1} + G_t, \quad (2)
$$

This equation differs from the usual capital accumulation equation in that current, rather than lagged government absorption increases the current stock of public infrastructure. Formally, as we will see below, this is done to nest the prototypical RBC model with production externality in the present framework, see, e.g. Farmer and Guo (1994). Economically, note that not only government investment, but also government consumption is modelled as infrastructure-augmenting, on the basic reasoning that "consumptive" components of government expenditures such as public education, enforcement of the law, external security and the like are clearly supportive of and hence complementary to private production. Given

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4Consequently, $S_t$ is not a state variable.
this broad view of infrastructure, the government wage bill, for instance, (as the larger part of
government consumption) clearly has an instantaneous infrastructure effect.

Assuming Ricardian equivalence, see e. g. Seater (1993), all government expenditure is tax
financed. The tax proceeds are modelled as a simple linear function of current output $Y$:

$$\tilde{G} = T \cdot Y$$  \hfill (3)

The log of the tax rate $\ln T$ is assumed to be a random walk without drift and with starting
value $\ln T$ at some distant point in the past.

Private production takes place under a Cobb-Douglas production function

$$Y = S^\phi \left( L, X \right)^\alpha K^{1-\alpha} \hfill (4)$$

where $0 < \alpha < 1$ and $\phi \geq 0$. Here, apart from the usual factors, production depends on the growth
component $X$, which is labor augmenting, and on the level of public infrastructure $S$. Since
the economy is inhabited by many agents, the individual producer takes $S$ as given and does
not recognize any link between $S$ and his own actions. In the aggregate, however, the economy
displays increasing returns to scale. In the special (but unrealistic) case of $d=1$ (complete
period depreciation of public infrastructure) this is most easily seen by substituting (2) and (3)
into (4) and solving for per-capita output:

$$d = 1 \quad \Rightarrow \quad Y = T^{\frac{\phi}{1-\phi}} \left( L, X \right)^\frac{\alpha}{1-\phi} K^{\frac{1-\alpha}{1-\phi}} \hfill (5)$$

Net exports $NX$ are assumed to depend linearly on output,

$$NX = x Y \hfill (6)$$

where $\ln z_t = \ln(1-x_t)$ is assumed to be a random walk without drift and with starting value zero
at some distant point in the past. The law of motion for the capital stock, finally, is given by

$$K = (1 - \delta) K_{t-1} + I_{t-1} \hfill (7)$$

where $I$ is private gross investment and $0 < \delta < 1$ is the constant rate of depreciation for
private capital.

Under these conditions, the resource constraint can be written as

$$C + I = (z - T) Y \hfill (8)$$

with $z_t = 1-x_t$.

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5 This way of reasoning follows an old line in the public finance literature, see e. g. Littmann (1957). Littmann
splits government expenses into transfers, government production and government consumption, where the latter
is very narrowly defined and basically comprises expenditures for representational purposes. While, admittedly,
one might argue that not all government absorption is used to provide public infrastructure, one might, on the
other hand, point out that even transfers are beneficial for private production. The German social security system,
for instance, is often credited for the relative absence of strikes and distributive quarrels in the German society.
Thus, under a wide definition of public infrastructure, government infrastructure investment might even be larger
than government absorption.
Solving (1) with respect to (4), (7), and (8) yields the following Euler-equations:

\[ C_t L_t^{\nu \psi} = \alpha (z_t - T_t) Y_t X_t^{\nu (1+\psi)} \]  
\[ E_t [C_{t+1}] = E_t \left[ \beta C_t \left( (1-\delta) + (z_{t+1} - T_{t+1}) \left( 1 - \alpha \right) \frac{Y_{t+1}}{K_{t+1}} \right) \right] \]  

To solve the model I use the technique proposed by King, Plosser and Rebelo (1988). Specifying the growth process as

\[ \ln X_{t+1} = \ln \gamma_X + \ln X_t + \varepsilon_{X,t+1} \]  

we can compute the steady state of the deterministic version of the economy (obtained by setting the variances of all disturbances equal to zero). It is a straightforward exercise to show that in the steady state hours \( L_t \) grow at rate \( \ln \gamma_X \), while consumption, investment, government expenditure, output and capital stock grow at rate \( \ln \gamma \) where \( \gamma \) is defined as

\[ \gamma := \gamma_X^{\alpha (1+\nu)/(\alpha - \psi)} \].

Note that unless \( \nu = 0 \), log hours are integrated of order one and cointegrate with other model variables in a way to be derived below. \( ^6 \)

I derive a non-growing economy by dividing each variable by its growth component. Denoting the growth-corrected variables by small letters we have

\[ l_t := \frac{L_t}{X_t^{\nu}}, \quad y_t := \frac{Y_t}{X_t^{\alpha (1+\nu)/(\alpha - \psi)}}, \text{ etc.} \]

Substituting (5) into (9) and writing this in terms of the transformed variables we get

\[ c_t = \alpha (z_t - T_t) T_t^{\frac{\nu (1+\psi)}{\alpha}} y_t^{-\frac{1-\phi(1+\psi)}{\alpha}} k_t^{-\frac{\alpha}{\alpha}} . \]  

Similarly, (10) and (8) can be rewritten as

\[ E_t [c_{t+1}] = E_t \left[ \frac{\beta c_t}{\gamma \exp \{ \varepsilon_{t+1} \} \left( (1-\delta) + (z_{t+1} - T_{t+1}) \left( 1 - \alpha \right) \frac{Y_{t+1}}{K_{t+1}} \right) \right] \]  

\( ^6 \)Recall that German hours per capita have a persistent negative trend; they are clearly instationary. This is basically due to a significant reduction of weekly hours worked pressed for by German trade unions. While integrated series have asymptotic properties incompatible with bounded time series like hours, they may nevertheless provide the best parsimonious description of such series in finite samples. In fact, a constant negative growth rate of hours is not incompatible with hours being nonnegative, the objection that this would imply virtually zero hours in some hundred years or so being of little practical relevance. Given the current (1975-1994) growth rate of hours of about -0.001 per quarter, working time would decrease by about 33% within 100 years from now. This is not an unreasonable scenario given the reduction in hours worked in the past 100 years. The reduction in per capita hours worked in West Germany in the last 35 years amounts already to about 30% of the 1960 level.
respectively. In (13) and (14), I use the shorthand notation $\varepsilon_t := \alpha(1+\nu)/(\alpha-\phi)\varepsilon_{x_t}$. Dropping time indices to denote steady-state-values, (12), (13) and (14) can be solved for steady-state output, consumption, and capital:

$$y = \frac{\phi}{1-T} (1-\alpha) \beta \frac{1-\gamma}{\gamma - \beta(1-\delta)} \left( \frac{\alpha(\gamma - \beta(1-\delta))}{(1-\beta)\gamma + \alpha\beta(\gamma - (1-\delta))} \right) \frac{\alpha}{\alpha-\phi}(1+\psi)$$

$$c = \frac{(1-\beta)\gamma + \alpha\beta(\gamma - (1-\delta))}{\gamma - \beta(1-\delta)}(1-T)y$$

$$k = \frac{(1-\alpha)\beta}{\gamma - \beta(1-\delta)}(1-T)y$$

Assuming that $\alpha > \phi$, a realistic assumption, and differentiating $c$ with respect to $T$, it is easy to see that steady-state consumption is maximized for $T=\phi$. Thus, if $T<\phi$ a tax increase will have a positive systematic effect on consumption, since the marginal benefit of increasing infrastructure outweighs the marginal tax burden. However, if $T>\phi$, shrinking government expenditures (and taxes) will tend to increase private consumption.

To solve equations (12) - (14) analytically, I compute a log-linear Taylor approximation of the system about its steady state. I use hatted variables to denote percent deviations from steady-state-levels of the non-growing economy, i. e.

$$\hat{y}_t := \ln y_t - \ln y, \quad \hat{c}_t := \ln c_t - \ln c, \quad \text{etc.}$$

The log-linear analogues of (12)-(14) are then given by

$$\hat{c}_t = \frac{1}{1-T} \hat{z}_t + \frac{\phi(1+\psi)}{\alpha} - \frac{T}{1-T} \hat{T}_t + \left( 1 - \frac{(1-\phi)(1+\psi)}{\alpha} \right) \hat{y}_t + \frac{(1-\alpha)(1+\psi)}{\alpha} \hat{k}_t, \quad (15)$$

$$\hat{c}_{t+1} = \hat{c}_t + \frac{\gamma - \beta(1-\delta)}{\gamma} \left( \hat{y}_{t+1} - \hat{k}_{t+1} - \frac{T}{1-T} \hat{T}_{t+1} - \hat{z}_{t+1} \right) - u_{t+1}, \quad (16)$$

$$= \frac{(1-\beta)\gamma + \alpha\beta(\gamma - (1-\delta))}{(1-\alpha)\beta} \hat{c}_t + \frac{\gamma - \beta(1-\delta)}{(1-\alpha)\beta} \left( \hat{y}_t + \frac{1}{1-T} \hat{z}_t - \frac{T}{1-T} \hat{T}_t \right), \quad (17)$$

Here $u_{t+1}$ is a zero mean error term which combines $\varepsilon_{x_{t+1}}$ and the Euler equation error incurred by replacing expected values by their realizations.
Solving (14) for $\dot{y}$ and substituting into (15) and (16) yields a system of two first order difference equations in the endogenous variables $\dot{c}_t$ and $\dot{k}_t$ as well as the forcing variables $\dot{T}_t$. The homogenous part of this system is of the general form

$$
\begin{pmatrix}
    a_{11} & a_{12} & b_{11} & 0 \\
    0 & a_{22} & b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
    \dot{c}_{t+1} \\
    \dot{k}_{t+1}
\end{pmatrix}
=
\begin{pmatrix}
    b_{11} & 0 \\
    b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
    \dot{c}_t \\
    \dot{k}_t
\end{pmatrix}.
$$

Thus the stability properties of the economy depend on the eigenvalues of the matrix $D := A^{-1}B$. In standard RBC-models one eigenvalue of $D$ is larger and the other smaller than unity in absolute value. In this case a stable solution to the difference equation exists for a linear subspace of the initial conditions. This means that for one starting value given (i. e. the predetermined value of $k_0$) the other starting value (i. e. $c_0$) is uniquely determined by requiring stability of the solution path. This is the so-called saddle-path stability of RBC-models.

If the eigenvalues of $D$ were both smaller than unity in absolute value, the saddle becomes a sink, meaning that for any pair of starting values the economy converges to the steady state. Since only $k_0$ is predetermined, $c_0$ can, in this case, assume any value. It can in particular be driven by sunspot expectations, which will necessarily be self-fulfilling, since any sunspot shock in consumption leads the economy on a sustainable convergent path to the steady state. The economy then displays equilibrium indeterminacy.

The algebra of the above difference equation is quite messy and I will therefore refrain from computing the eigenvalues of $D$ in terms of the structural parameters. Rather, I follow Wen (1996), who points out that a necessary condition for $D$ to have both eigenvalues smaller than one in absolute value is that the determinant of $D$ is smaller than one in absolute value. (Clearly, this condition is not sufficient). The determinant of $D$ can be derived as

$$
\det D = \frac{1}{\beta} \left[ 1 - \frac{\phi(1+\psi)(\gamma - \beta(1-\delta))}{\alpha \beta (1-\delta) - \gamma (1-\phi)(1+\psi)} \right] = \frac{1}{\beta} [1 - \chi].
$$

This expression is smaller than one only if $\chi$ is positive. Thus one obvious necessary condition for equilibrium indeterminacy is $\phi > 0$, i. e. the existence of increasing returns to scale in aggregate private production. A further necessary condition for sunspot fluctuations is $\psi < -1$, for if $\psi$ were smaller than $-1$, then the numerator of $\chi$ would be negative and the denominator positive, i. e. $\chi$ would be negative. These conditions will be very helpful in the empirical analysis below.

### 3. Data and Regression Equations

Quarterly data for the main economic aggregates of the West German economy are available from 1960 to 1994, since the system of national accounts had fictitiously separated West and East Germany from 1990 until the end of 1994. However, data prior to 1975 do not lend
themselves easily to an analysis of the kind intended here, since in this earlier sample the investment share in GDP is not constant but higher than today and gradually decreasing. Likewise, the consumption share is lower than today and gradually increasing. These phenomena are probably due to accumulating productive capital after the severe losses incurred in World War II. Thus the data do not allow for a steady state interpretation prior to 1975 and therefore I will use a sample of 80 observations ranging from 1975.1 to 1994.4.

Further, the share of government absorption in GDP is negatively trended in this sample, while government transfers (as a percentage of GDP) have a (roughly offsetting) positive trend. The share of net exports in GDP, incidentally, has roughly the same positive trend. Prior to 1990 the reason for this can at least partially be traced to the rising importance of transfers to the European Union: First, these transfers somewhat limited the scope for government absorption in Germany. Second, however, in as much as the European Union used these transfers for absorption of German goods (for instance by intervening in agricultural markets), the transfers artificially increased German exports. Clearly, such exports should actually be counted as government absorption as well.

In 1990 and the following years, the German government transferred huge amounts of money to East Germany. These were counted as transfers to the rest of the world in the system of national accounts that still separated both parts of Germany. To the extent to which East German regional governments used these transfers to buy goods and services in West Germany, West German exports increased. Again, it would be better to count these exports as government absorption.

There is no easy correction for these distorting developments in the system of national accounts. As an admittedly crude correction I have added net government transfers to the rest of the world to government absorption at home and subtracted these transfers from net exports. Better methods may exist, but the one I have chosen serves its purposes well, since in their new definitions government absorption and net exports have roughly constant shares in GDP.

I have used (seasonally adjusted) data from the OECD Quarterly National Accounts, since the OECD also provides capital stock data, although only in a yearly periodicity. I have used the data on quarterly private investment to construct quarterly capital stock data, assuming that each year's depreciation is distributed evenly on the four quarters. Since the RBC-model is of a representative agent type, I divide all series by a population series provided by the Deutsches Institut für Wirtschaftsforschung. The latter institution also provided (seasonally adjusted) data on hours worked and on the real wage.

We are now interested in estimation of the parameters $\phi$ and $\psi$. For this purpose we need regression equations that can be estimated with standard methods. Since levels of the model variables are integrated of order one, it is of prime interest to derive the cointegration properties of the system. Transforming the hatted variables back into observables, equations (16) and (17) become:

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7The series used is total population ("Wohnbevölkerung").
8Note in particular the relation $\hat{\xi}_t = \ln (Y_t - NX_t) - \ln Y_t$. 
\[
\Delta \ln C_t = c_{10} + \frac{\gamma - \beta (1 - \delta)}{\gamma} \left[ \frac{1}{1-T} \ln(Y_t - NX_t) - \frac{T}{1-T} \ln G_t - \ln K_t \right] + \nu_t
\]
\[
= c_{10} + c_{1K} \ln K_t + c_{1NX} \ln(Y_t - NX_t) + c_{1G} \ln G_t + \nu_t
\]
(18)

\[
\Delta \ln K_{t+1} = c_{20} + \frac{\gamma - (1 - \delta)}{\gamma} \ln C_t + \frac{\gamma - \beta (1 - \delta)}{\gamma} \left[ \frac{1}{1-T} \ln(Y_t - NX_t) - \frac{T}{1-T} \ln G_t - \ln C_t \right]
\]
\[
= c_{20} + c_{2K} \ln K_t + c_{2C} \ln C_t + c_{2NX} \ln(Y_t - NX_t) + c_{2G} \ln G_t
\]
(19)

Here \(c_{10}\) and \(c_{20}\) are constants and \(\nu_t\) is the Euler equation error. Since the left hand side of (18) is stationary, so must be the right hand side, and therefore we see that \(\ln K_t\) cointegrates with \(\ln(Y_t - NX_t)\) and \(\ln G_t\). Using this fact we learn from equation (19) that \(\ln C_t\) cointegrates with \(\ln K_t\) and that the cointegrating vector is \((1, -1)\)'.

Proceeding similarly with equation (15) and collecting the constant terms in \(\tilde{c}_{30}\) results in
\[
\ln C_t - \frac{1}{1-T} \ln(Y_t - NX_t) + \frac{T}{1-T} \ln G_t = \tilde{c}_{30} + \frac{1 + \psi}{\alpha} \left[ (1 - \alpha) \ln K_t - \ln Y_t + \phi \ln G_t + \alpha(1 + \nu) \ln X_t \right]
\]
(20)

an equation that cannot be estimated easily, since \(\ln X_t\) is not observed. Yet we see from (20) that \(\ln Y_t\) cointegrates with \(\ln K_t\), \(\ln G_t\), and \(\ln X_t\) since the right hand side is a cointegrating combination. Solving (5) for \(\ln X_t\) and substituting into (20) we get
\[
\ln L_t = \frac{1}{(1 + \psi)(1 + \nu)} \left[ \tilde{c}_{30} + \frac{1}{1-T} \ln(Y_t - NX_t) - \frac{T}{1-T} \ln G_t - \ln C_t \right]
\]
\[
+ \frac{\nu}{\alpha(1 + \nu)} \left[ \ln Y_t - (1 - \alpha) \ln K_t - \phi \ln G_t \right]
\]
\[
= c_{30} + c_{3K} \ln K_t + c_{3C} \ln C_t + c_{3Y} \ln Y_t + c_{3NX} \ln(Y_t - NX_t) + c_{3G} \ln G_t
\]
(21)

which implies that for \(\nu \neq 0\) \(\ln L_t\) is cointegrated with \(\ln Y_t\), \(\ln K_t\), and \(\ln G_t\), while for \(\nu = 0\) \(\ln L_t\) is stationary.

In a system consisting of the six variables \(\ln K_t\), \(\ln C_t\), \(\ln Y_t\), \(\ln(Y_t - NX_t)\), \(\ln G_t\), and \(\ln L_t\) and driven by the three independent stochastic trends \(\ln X_t\), \(\ln z_t\), and \(\ln T_t\) we have thus found three independent cointegrating vectors. These form a basis of the cointegration space and are, for convenience, summarized in Table 1.
In the empirical analysis, the first step taken is a check of the long-run implications of the model, i.e. at its cointegration properties. I will use Johansen's (1988, 1995) general maximum-likelihood-method to test for multiple cointegration in the system of the six variables $\ln K_t$, $\ln C_t$, $\ln Y_t$, $\ln (Y_t - NX_t)$, $\ln G_t$, and $\ln L_t$. The specification allows for a linear trend in the data by allowing for a constant in first differences and in the cointegrating relationships. An additional linear trend in the cointegrating relationships is not permitted, as the model supposes that all trend components have been correctly specified.

The Johansen-tests have been carried out by Eviews, version 2.0. The program readily provides Q-statistics for individual residual series, but does not provide procedures to test for residual whiteness in a system perspective. I have therefore tested for autocorrelation in the six-dimensional error process by using the multivariate version of the Ljung-Box-Q-test as described, e.g., by Lütkepohl (1991). The procedure was written by myself in GAUSS, version 3.2, and uses the correct number of freedoms taking into account the cointegration restriction on the VAR. The lag-length tested for is four years (16 quarters).

Table 3 gives the results of the Johansen-Tests for the six-dimensional system, estimated with two lags in first differences. Residuals are approximately white noise with this lag specification, since the Q-statistic is not significant at the 5%-level. The implied P-value 9% is rather low, however, indicating that some evidence of autocorrelation may still exist. The consequences of a longer lag-length will be explored below.

9In this respect, it differs from the formula given in Lütkepohl, p. 152, since Lütkepohl introduces this test in the context of an unrestricted VAR.
We see from Table 2 that the hypothesis of no cointegrating vector is rejected at the 1%-level and the hypothesis of at most one cointegrating vector is rejected at the 5% level. The hypothesis of at most two cointegrating vectors can be accepted at the 5%-level, but the value of the test statistic is quite close to the 5%-critical value and hence, since the model postulates the existence of three cointegrating vectors, I decided to reject the hypothesis of not more than two vectors as well. The hypothesis of at most three cointegrating vectors, finally, cannot be rejected at a reasonable level of significance and is thus accepted.

Turning to the estimated long-run coefficients, recall that under the normalization chosen the coefficients of \( \ln(Y_t - NX_t) \), \( \ln G_t \), and \( \ln L_t \) should be equal in the first and second row of the lower panel of Table 2. This property is not fulfilled for the point estimates, but the standard errors are quite large so that a formal test-statistic for the hypothesis of equality might possibly not be significant.

However, I will not use such a test for in the six-dimensional system. The reason is that the results given in Table 2 are not particularly stable with respect to the lag length chosen. For the same system and lag length three (in differences) we obtain, cf. Table 3, evidence for only one cointegrating vector. Moreover, the point estimates cannot be considered close to those in Table 2. While the Q-statistic indicates some progress as to the whiteness of the residuals, it remains unclear, whether the system with two lags is underparameterized (and coefficient
estimates are biased) or whether the system with three lags is overparameterized (and coefficient estimates are inefficient). General experience, however, often casts doubt on the reliability of inference in high-dimensional VARs, so that in the next step I will reduce the dimension of the VAR by exploiting the zero restrictions given in Table 1.

Table 3

Johansen-Test for Cointegration: 3 Lags

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Trace-Stat.</th>
<th>5% Crit. Val.</th>
<th>1% Crit. Val.</th>
<th>H0: Dim. of coint. space</th>
<th>Q(16) (P-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.41</td>
<td>100.2</td>
<td>94.2</td>
<td>103.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>60.1</td>
<td>68.5</td>
<td>76.1</td>
<td>1</td>
<td>553 (0.247)</td>
</tr>
<tr>
<td>0.20</td>
<td>29.4</td>
<td>47.2</td>
<td>54.5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Normalized Cointegrating Coefficients: 3 Cointegrating Equations

<table>
<thead>
<tr>
<th>lnK_t</th>
<th>lnC_t</th>
<th>lnY_t</th>
<th>ln(Y_t-NX_t)</th>
<th>lnG_t</th>
<th>lnL_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.67 (1.07)</td>
<td>1.54 (0.91)</td>
<td>4.80 (1.53)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1.21 (0.18)</td>
<td>0.46 (0.15)</td>
<td>0.63 (0.26)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-0.77 (0.28)</td>
<td>0.35 (0.23)</td>
<td>1.32 (0.39)</td>
</tr>
</tbody>
</table>

I first check the bivariate system of consumption and the capital stock for cointegration. It turns out that it is difficult to find an appropriate lag length specification since, apparently, capital adjustment is rather slow. Thus, in the absence of other conditioning variables, a lag length of nine quarters seems necessary to obtain reasonable results. Table 4 shows that in such a specification, consumption and capital are cointegrated at the 1% level and the estimated coefficient for consumption deviates from the theoretically expected value of -1 by about one standard deviation. These results basically confirm the first cointegrating vector in Table 1.
Table 4
Johansen-Test for Cointegration: 9 Lags

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Trace-Stat.</th>
<th>5% Crit. Val.</th>
<th>1% Crit. Val.</th>
<th>H0: Dim. of coint. space</th>
<th>Q(16) (P-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>22.5</td>
<td>15.4</td>
<td>20.0</td>
<td>0</td>
<td>126 (0.595)</td>
</tr>
</tbody>
</table>

Normalized Cointegrating Coefficients: 1 Cointegrating Equation
(standard errors in parentheses)

<table>
<thead>
<tr>
<th>lnKt</th>
<th>lnCt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.34 (0.35)</td>
</tr>
</tbody>
</table>

Next I analyze the trivariate system consisting of consumption, output less net exports, and government absorption, which, according to the model, should also be characterized by one cointegrating vector. Using three lags in the Johansen procedure, that the hypothesis of no cointegration can indeed be rejected (at the 1% level), while the hypothesis of one cointegrating vector is accepted, cf. Table 5.

Table 5
Johansen-Test for Cointegration: 3 Lags

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Trace-Stat.</th>
<th>5% Crit. Val.</th>
<th>1% Crit. Val.</th>
<th>H0: Dim. of coint. space</th>
<th>Q(16) (P-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>37.3</td>
<td>29.7</td>
<td>35.7</td>
<td>0</td>
<td>126 (0.595)</td>
</tr>
<tr>
<td>0.11</td>
<td>13.2</td>
<td>15.4</td>
<td>20.0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Normalized Cointegrating Coefficients: 1 Cointegrating Equation
(standard errors in parentheses)

<table>
<thead>
<tr>
<th>lnCt, ln(Yt-NXt), lnGt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>-1.52 (0.17)</td>
</tr>
</tbody>
</table>
The coefficient estimates for ln($Y_t-NX_t$) and ln$G_t$ independently imply estimates of $T$, the steady state share of government absorption in GDP. These estimates are surprisingly close; they amount to 0.34 computed from the coefficient of ln($Y_t-NX_t$) and 0.33 computed from the coefficient of ln$G_t$. These values (obtained from second moments) are somewhat too high if compared with the time series $G_t/Y_t$: The mean, i.e. the first moment, is 0.24 and the series actually never exceeds 0.28, see Figure 1. To see whether the hypothesis of cointegration can still be maintained with lower values of $T$, I set $T=0.25$ and subject the series ln$C_t-1.33$ ln($Y_t-NX_t$) + 0.33 ln$G_t$ to an Augmented Dickey-Fuller-test. With a constant and three lags included the test statistic is -3.31, i.e. the series is clearly stationary. (The 5% critical value is -2.90). Thus I conclude that the three series are cointegrated as predicted by the model and that the cointegrating vector is compatible with $T=0.25$. Note that the thereby implied coefficients of the cointegrating vector do hardly differ by more than one standard error from the above estimates.

Figure 1

![Graph of Share of Government Absorption in GDP](image)

To verify the existence of the third cointegrating vector from Table 1, I now apply the Johansen test to the four-dimensional system consisting of ln$K_t$, ln$Y_t$, ln$G_t$, and ln$L_t$. With three lags the hypothesis of no cointegrating vector cannot be rejected at the 5%-level, but, again, the test statistic is very close to the respective critical value. Thus, in view of the model implications, I stick to my earlier decision to accept the existence of one cointegrating vector, cf. Table 6.
The coefficients of the cointegrating vector provide information about the structural parameters $\alpha$ and $\phi$. Taking into account that $\nu$ is merely a trend parameter for hours which is easily estimated to be around $-0.2$ from mean growth rates, the coefficients of $\ln K_t$ and $\ln L_t$ independently imply values of $\alpha=0.64$ and $\alpha=0.55$. The first of these is quite a reasonable value for the labor share in total income, while the second is probably a little low. The difference is not dramatic, however. The coefficient of $\ln G_t$ should equal $\phi$ and should thus give us an idea of the degree of externality in aggregate production. While a positive estimate was to be expected from the model, a value of almost 0.6 deems too high. Quite possibly, this coefficient is subject to some estimation bias as there may still be some autocorrelation in the residuals, cf. the P-value of the Q-statistic which is only barely above 5%.

Thus it seems that the long run implications of the model are roughly fulfilled in the data as far as the cointegration properties are concerned. However, coefficient estimates are somewhat unsatisfactory. In order to derive more reliable estimates of the structural parameters, I will hence use a different approach.

The key parameters of interest are certainly $\phi$ and $\psi$ since they tell us whether the necessary conditions for sunspot fluctuations are fulfilled. The equation to be estimated is hence equation (21), in which both of these parameters appear. Unfortunately, identification is not trivial, since there is a nonlinearity in the parameters. Using an estimate of $T$, however, we can rewrite equation (21) in a form that makes it easy to retrieve $\phi$ and $\psi$ from regression coefficients. Using the value of $T=0.25$ that was found to accord with cointegration properties of the data, equation (21) becomes
\[
\ln L_t = \frac{1}{(1+\psi)(1+\nu)} \left[ \tilde{c}_{30} + \frac{4}{3} \ln(Y_t - NX_t) - \frac{1}{3} \ln G_t - \ln C_t \right] \\
+ \frac{V}{\alpha(1+\nu)} \left[ \ln Y_t - (1-\alpha) \ln K_t - \phi \ln G_t \right] \\
= \tilde{c}_{30} + \tilde{c}_{3c} \ln EC_t + \tilde{c}_{3y} \ln Y_t + \tilde{c}_{3k} \ln K_t + \tilde{c}_{3g} \ln G_t 
\]  

(22)

and we have

\[
\phi = -\frac{\tilde{c}_{3g}}{\tilde{c}_{3y}}, \quad \psi = \frac{\tilde{c}_{3y} + \tilde{c}_{3k} - 1}{\tilde{c}_{3c}} - 1.
\]

In order to cope with the simultaneous equations bias, I estimate (22) by instrumental variables (IV). Lagged real wages are quite powerful in explaining current consumption, so I construct an instrument for \(\ln C_t\) by regressing \(\ln C_t\) on log real wages lagged one period and a linear trend. All other variables I just lag by one period to form instruments. The regression results are given in Table 7. The implied values of the structural parameters are \(\alpha = 0.67\), \(\phi = 0.22\), and \(\psi = -1.04\).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNST</td>
<td>-0.57</td>
<td>0.24</td>
<td>-2.40</td>
<td>0.019</td>
</tr>
<tr>
<td>(EC_t)</td>
<td>-0.40</td>
<td>0.18</td>
<td>-2.23</td>
<td>0.029</td>
</tr>
<tr>
<td>(\ln Y_t)</td>
<td>1.52</td>
<td>0.49</td>
<td>3.09</td>
<td>0.000</td>
</tr>
<tr>
<td>(\ln K_t)</td>
<td>-0.50</td>
<td>0.11</td>
<td>-4.40</td>
<td>0.003</td>
</tr>
<tr>
<td>(\ln G_t)</td>
<td>-0.33</td>
<td>0.08</td>
<td>-4.21</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 7

Instrumental Variables Results: Dependent Variable is \(\ln L_t\)

If we take these point estimates seriously, then the IV-results imply that there is no scope for sunspot fluctuations in Germany, since \(\psi\) is smaller than -1. But obviously, this conclusion would be premature, as the implied value of \(\psi\) is remarkably close to -1. I will therefore compute the maximum eigenvalue of matrix \(D\) for varying configurations of parameters.

Matrix \(D\) depends on the six structural parameters \(\alpha, \beta, \gamma, \delta, \phi,\) and \(\psi\). A discount factor \(\beta = 0.997\) implies a real interest rate of roughly 3% and should thus be realistic, the parameter does not allow for much variation. The parameter \(\gamma\) is determined by the mean growth rates of the main economic aggregates and does not permit much variation either; with 2% real GDP-growth \(\gamma\) must equal 1.005. The rate of depreciation, finally, was implicitly determined by
constructing quarterly capital stock data; it equals 0.005 or 2% per year. Therefore, there is not much scope for variation in these parameters and hence the stability properties of the model economy essentially depend on $\alpha$, $\phi$, and $\psi$.

Table 8 displays the maximum eigenvalue (in absolute terms) of $D$ for a rather comprehensive selection of parameter values, where $\beta$, $\gamma$, and $\delta$ have been "calibrated" in the way described above: In the first row of Table 8 I give the maximum eigenvalue of $D$ for the estimated values of $\alpha$, $\phi$, and $\psi$; this eigenvalue is necessarily larger than one since $\psi$ is smaller than -1. Successively increasing $\psi$ still results in eigenvalues larger than one, until $\psi$ is as high as -0.55. Thus, given the other values are correct, sunspot fluctuations are possible only in the case of a strongly biased estimate of $\psi$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\phi$</th>
<th>$\psi$</th>
<th>Max. Eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-1.05</td>
<td>1.0054</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.95</td>
<td>1.0013</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.85</td>
<td>1.0011</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.75</td>
<td>1.0007</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.65</td>
<td>1.0002</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.55</td>
<td>0.9994</td>
</tr>
<tr>
<td>0.97</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0009</td>
</tr>
<tr>
<td>0.87</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0008</td>
</tr>
<tr>
<td>0.77</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0007</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0005</td>
</tr>
<tr>
<td>0.57</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0002</td>
</tr>
<tr>
<td>0.47</td>
<td>0.22</td>
<td>-0.70</td>
<td>0.9996</td>
</tr>
<tr>
<td>0.67</td>
<td>0.02</td>
<td>-0.70</td>
<td>1.0013</td>
</tr>
<tr>
<td>0.67</td>
<td>0.12</td>
<td>-0.70</td>
<td>1.0009</td>
</tr>
<tr>
<td>0.67</td>
<td>0.22</td>
<td>-0.70</td>
<td>1.0005</td>
</tr>
<tr>
<td>0.67</td>
<td>0.32</td>
<td>-0.70</td>
<td>1.0001</td>
</tr>
<tr>
<td>0.67</td>
<td>0.42</td>
<td>-0.70</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

In the second and third panel of Table 8 I assume that the our estimate of $\psi$ is subject to a strong negative bias and that the true value of $\psi$ is -0.7. Varying $\alpha$, we see that only in the case of a labor share of less than 50% would the economy display sunspot fluctuations. Such a low value of $\alpha$ is certainly not realistic. Similarly, setting $\alpha=0.67$ again, we would need a value for $\phi$ larger than 0.32 to push both eigenvalues below one. But such a large value of $\phi$ is
unlikely to hold in reality, as this would imply that the aggregate production function is homogenous of degree \(1/(1-\phi)##1.5 or more!

Thus, given our regression results and the above stability analysis, it seems rather unlikely that the non-keynesian phenomena observed in the 1980s are to be attributed to sunspot fluctuations. Rather, the coefficient estimates suggest an easy and straightforward systematic explanation for the effects of reduced government absorption: Note that the estimated value for \(\phi\) is 0.22 which is lower than any of the observed values for \(T_i\) in the sample. This suggests that a government policy aiming at a permanent reduction of government absorption, i. e. an attempt to lower the steady-state value \(T\), increases the steady state value of consumption. (Recall that steady-state consumption is maximized for \(T=\phi\). In this view, the increase in consumption is due to a government policy that makes resources with low marginal infrastructure productivity available for private use.

5. Conclusions

At several occasions, the 1980’s witnessed non-keynesian effects of fiscal contractions. Most notably, cuts in government spending in Ireland, Denmark and Germany are known to have coincided with increases in private consumption spending. Self-fulfilling expectations about the effects of stabilization policies may explain these and a diversity of opposing experiences. This paper formulates a real business cycle model with equilibrium indeterminacy, which allows for sunspot fluctuations as well as for systematic consumption effects of government policy in either direction. Cointegration tests and Euler equation estimates suggest that the model is approximately in accord with German data. Parameter estimates imply that the non-keynesian experiences are not due to self-fulfilling expectations but to the productivity effects of government-provided infrastructure.

References:


